XU

(How to Construct a Hedonometer)

Loren K. Fryxell

City St George's, University of London

loren.fryxell@citystgeorges.ac.uk

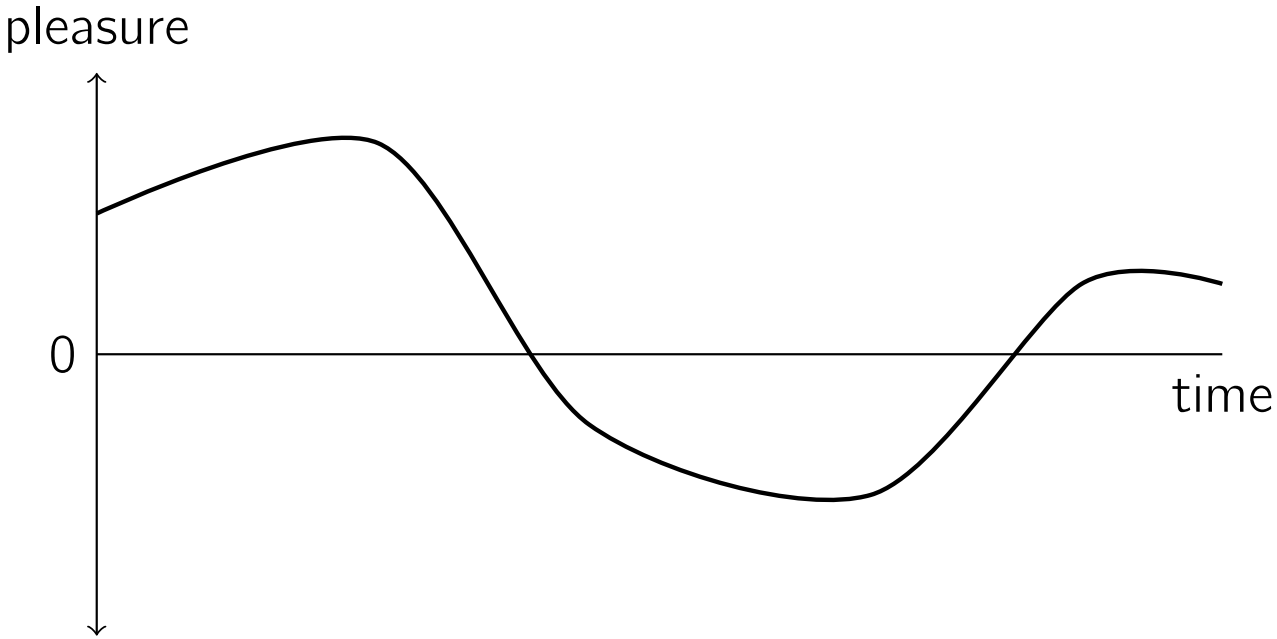
November 5, 2025

City Research Day, 2025

How to Construct a Hedonometer

Edgeworth's Hedonometer

A long time ago, Edgeworth proposed the idea of a *hedonometer*



Constructing the Hedonometer

I think we can all agree this is a very intuitive idea

Even so, the idea doesn't tell us what the y axis really is

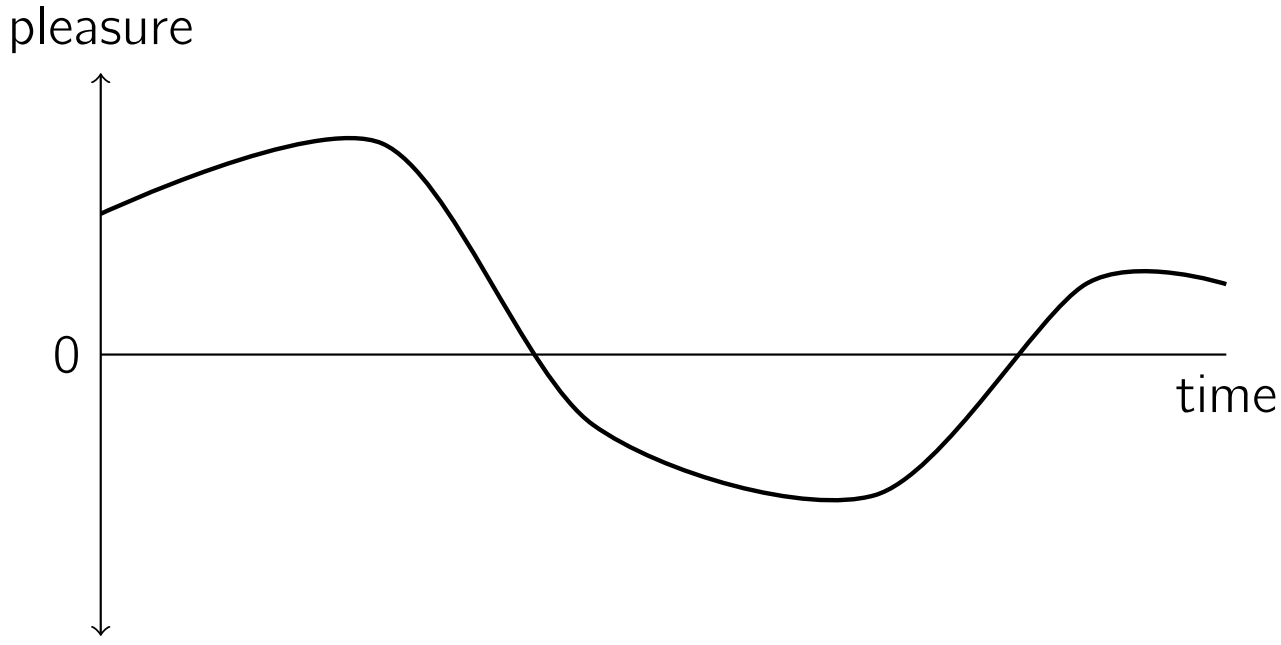
What is pleasure and how can we measure it?

Edgeworth proposed the *idea* of a hedonometer, but he didn't tell us how to *actually construct* one

That's where I come in

I will tell you how to construct a hedonometer

Day at Work Example



To fix ideas, let's consider a day at work

We would like to construct a hedonometer for this experience

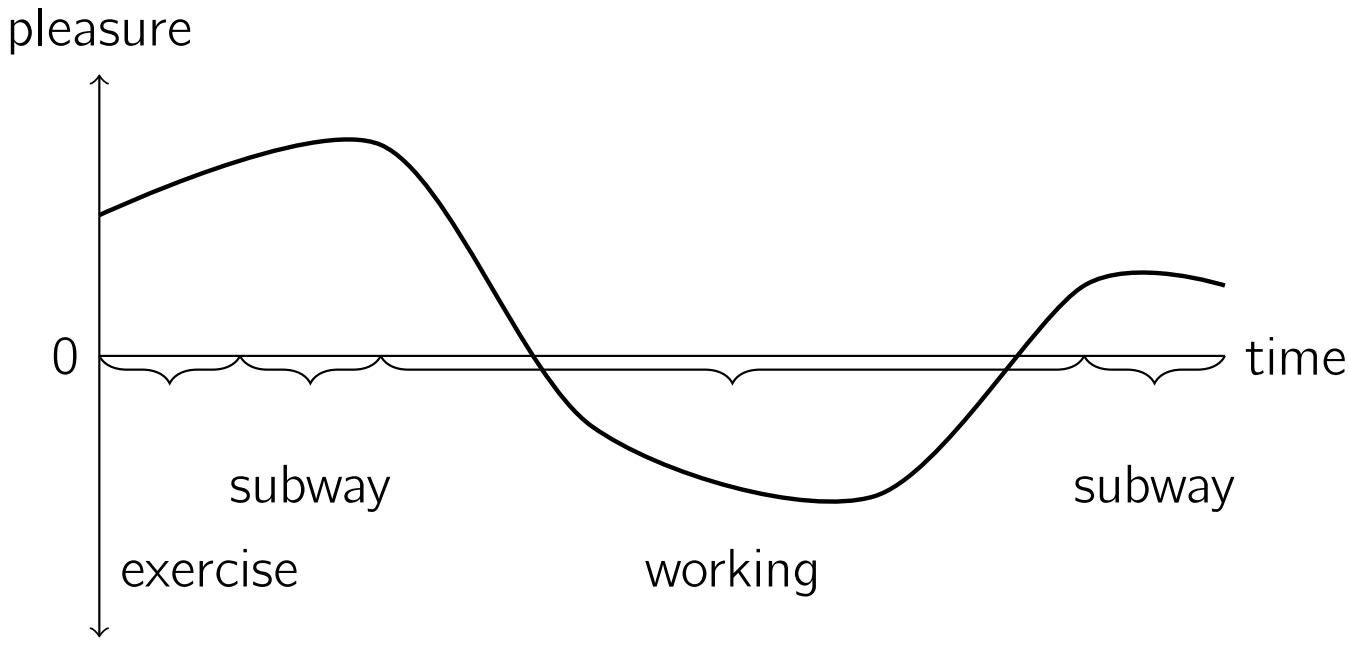
Day at Work Example

This day at work comprises many (sub)experiences

- an hour of exercise in the morning
- an hour on the subway in the morning
- several hours working in the office
- an hour of the subway in the evening

Each experience will be represented by an *interval*

Experiences are Intervals



Main Result in Words

If you have preferences over all such intervals, then I can back out precisely what your hedonometer must be

In other words,

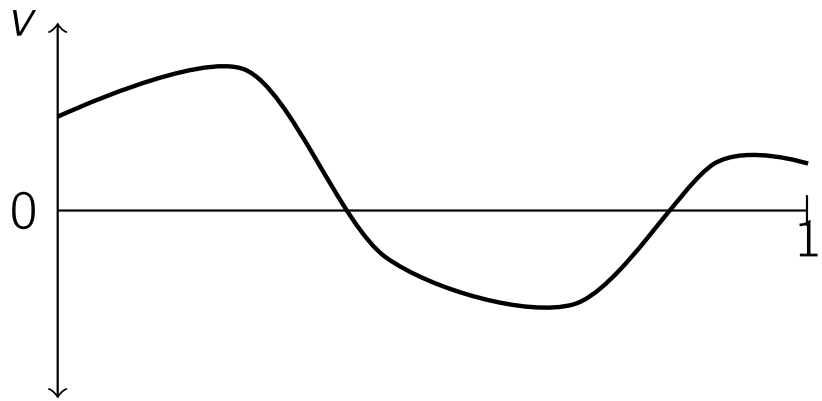
- you tell me which of any two experiences (intervals) you prefer
- and I will construct your hedonometer

Main Result in Math

Theorem. Three axioms hold if and only if there exists a function $v : [0, 1] \to \mathbb{R}$ such that for any two intervals I and J,

$$I \succeq J \iff \int_I v(t)\,\mathrm{d}t \geq \int_J v(t)\,\mathrm{d}t.$$

v is the hedonometer (instantaneous pleasure over time) and is unique up to a positive scalar.



Axioms

There are three axioms:

1. Rationality
2. Monotonicity
3. Continuity

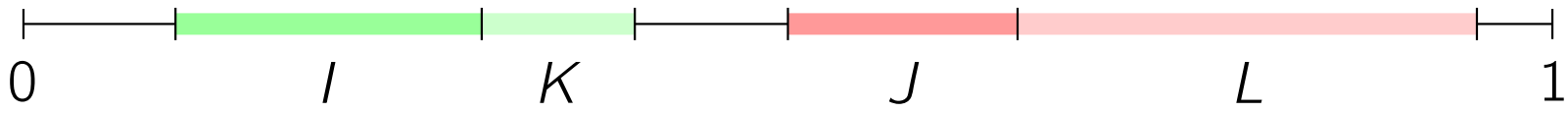
Axiom 1. Rationality

Axiom 1. An individual has complete and transitive preferences $\succeq$ over $\mathcal{I}$.

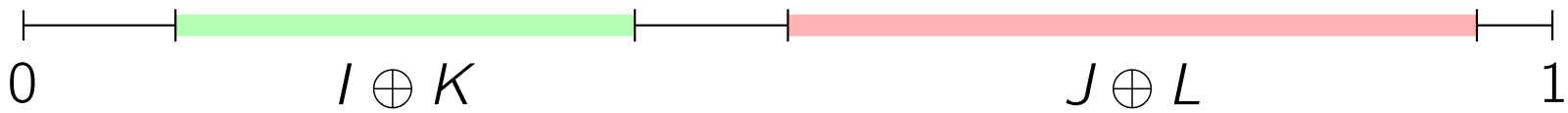
Axiom 2. Monotonicity

Suppose I and K are adjacent, and J and L are adjacent

If $I \succeq J$ and $K \succeq L$



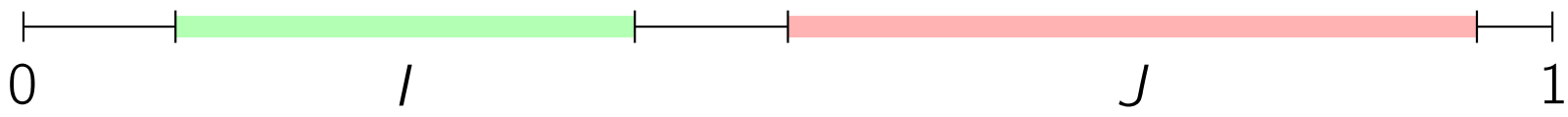
then $I \oplus K \succeq J \oplus L$



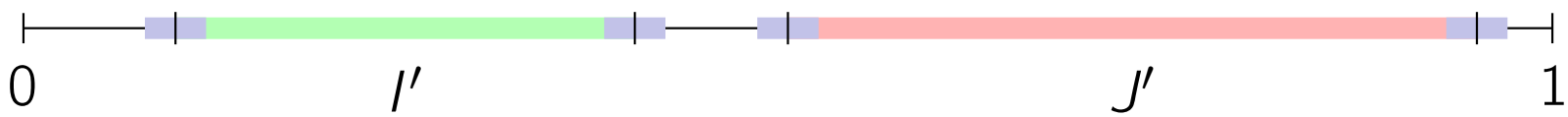
Axiom 3. Continuity

Small changes in an interval result in small changes in preference

If $I \succ J$



then $I' \succ J'$



Constructing the Hedonometer

The proof is constructive

I will now show you how to construct the XU representation, i.e., the hedonometer

We will do so for an experience which is everywhere non-negative (the full construction requires doing so for all such segments and stitching them together afterwards)

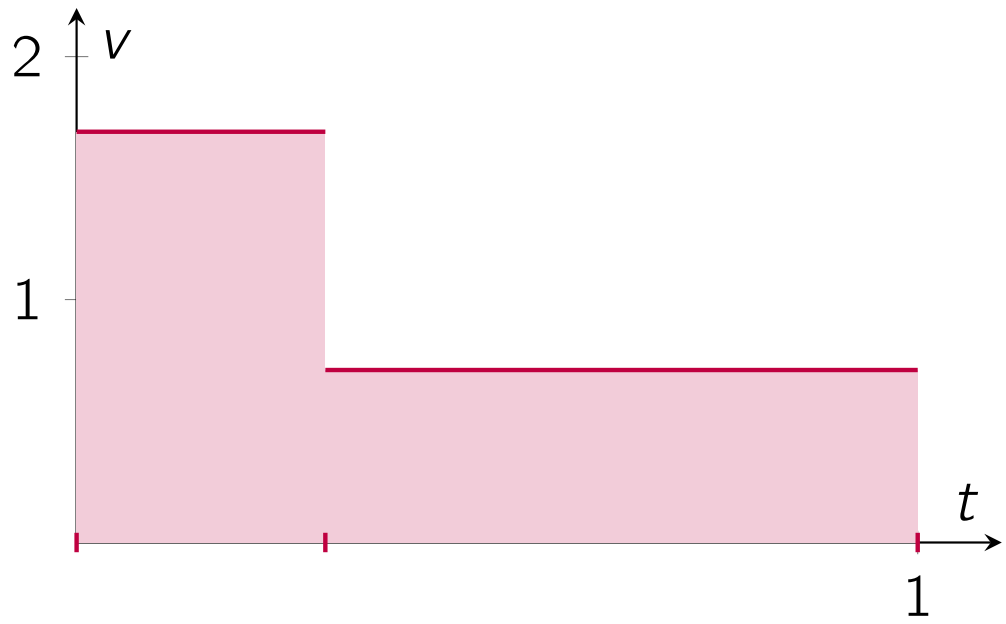
Constructing the Hedonometer

Suppose we have an experience which is everywhere non-negative (which means each slice of this experience would be preferred to having no experience)

Normalize the utility of the full experience to 1

Constructing the Hedonometer

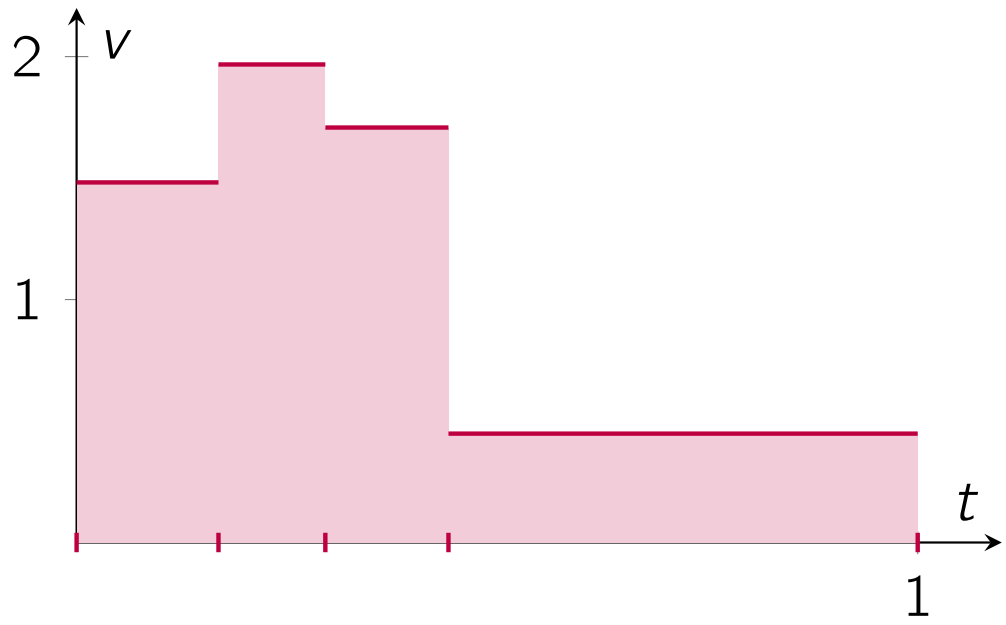
Cut it in half ($n = 1$)



Find the dividing point such that the first segment is indifferent to the second segment. Assign each utility $\frac{1}{2}$.

Constructing the Hedonometer

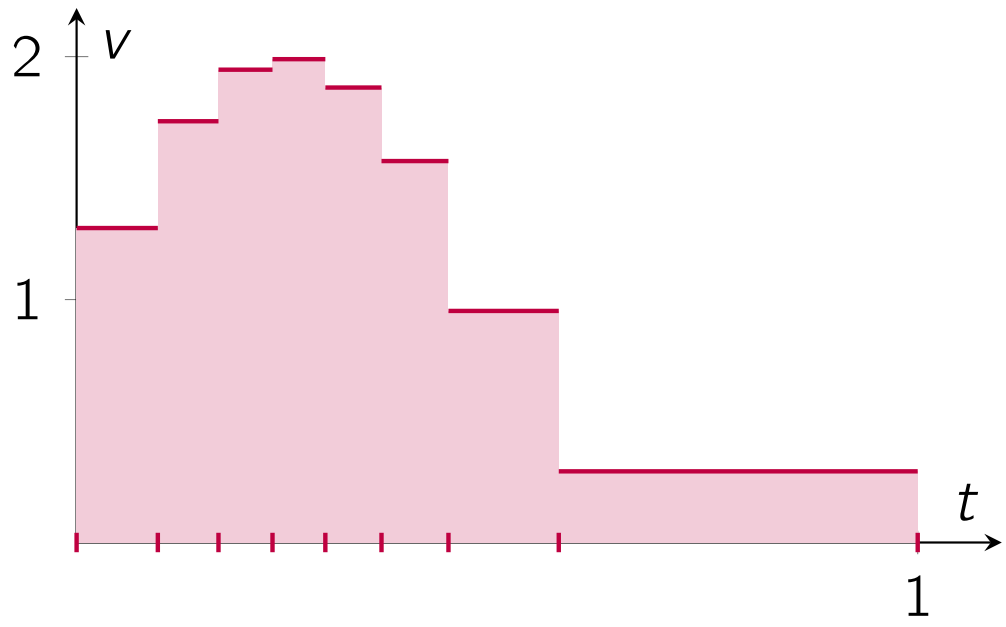
Cut each in half again ($n = 2$)



Do this again for each of the previous segments. Assign each utility $\frac{1}{4}$.

Constructing the Hedonometer

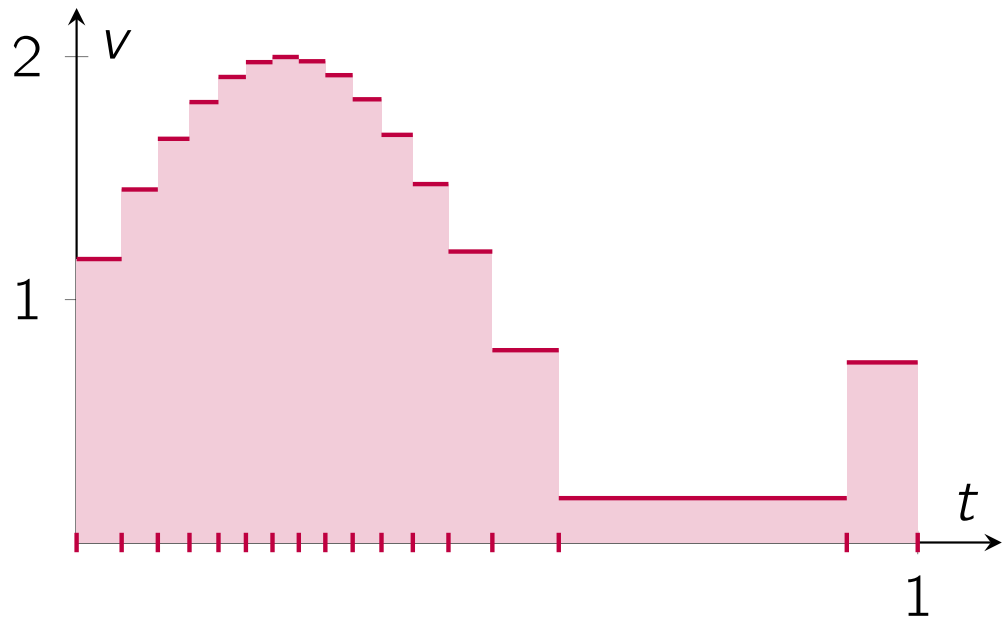
And again ($n = 3$)



Do this again for each of the previous segments. Assign each utility $\frac{1}{8}$.

Constructing the Hedonometer

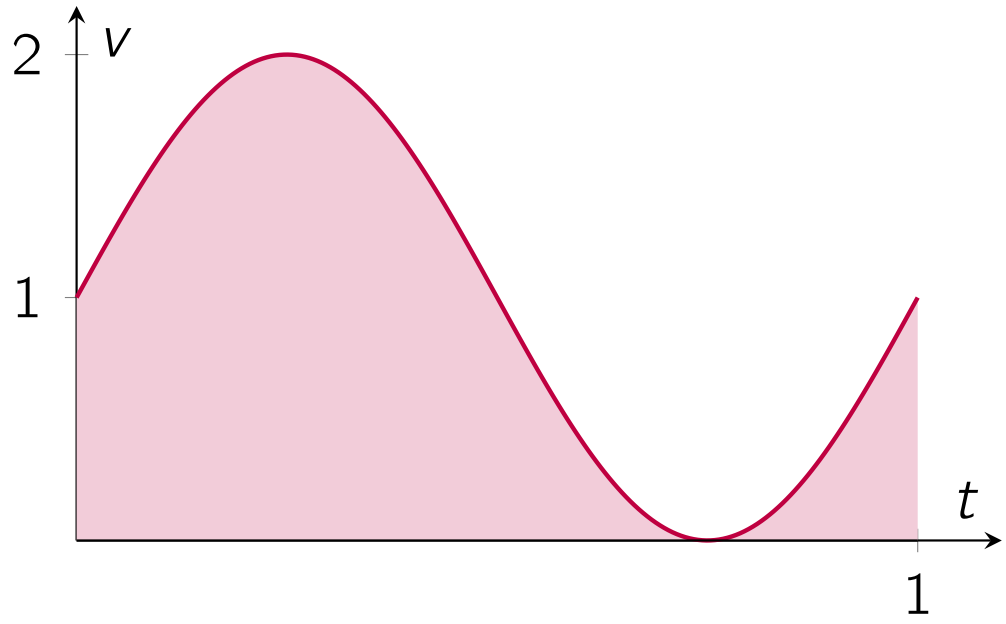
And again ($n = 4$)



Do this again for each of the previous segments. Assign each utility $\frac{1}{16}$.

Constructing the Hedonometer

And again and again.. ($n = \infty$)



Constructing the Hedonometer

QED

Thank you for listening!

Questions, comments, or concerns?